lead to

$$\int_0^\pi \frac{d\theta}{(7+\cos\theta)^2} = \frac{7\sqrt{3}}{576}\pi$$

so that

$$V = \frac{80\pi}{3}(8 - 3\sqrt{3}) \text{ cu. in.}$$

$$V \doteq 234.894,585,349,6 \text{ cu. in.}$$

Also solved by G. A. KNAPP, C. N. SCHMALL, GEORGE PAASWELL, H. N. CARLETON, O. S. ADAMS, and the PROPOSER.

408. Proposed by CLIFFORD N. MILLS, Brookings, South Dakota.

The ellipse $(x^2/81) + (y^2/16) = 1$ is revolved around the y-axis. Find the area of the surface generated.

SOLUTION BY CLYDE S. ATCHISON, Washington and Jefferson College.

From the given equation, we have

$$x = \frac{9}{4}\sqrt{16-y^2};\quad dx = -\frac{9y\,dy}{4\sqrt{16-y^2}};\quad \text{and}\quad ds = \frac{1}{4}\sqrt{\frac{256+65y^2}{16-y^2}}\cdot dy.$$

Then the area of the surface generated is

$$2\pi\int_{y=-4}^{y=4} x\cdot ds = 2\pi\int_{-4}^{4}\left(\frac{9}{4}\sqrt{16-y^2}\right)\left(\frac{1}{4}\sqrt{\frac{256+65y^2}{16-y^2}}\right)dy = \frac{9\pi}{8}\int_{-4}^{4}\sqrt{256+65y^2}\cdot dy$$

$$= \frac{9\pi\sqrt{65}}{16}\left[y\cdot\sqrt{y^2+\frac{256}{65}} + \frac{256}{65}\log\left(y+\sqrt{y^2+\frac{256}{65}}\right)\right]_{-4}^{4}$$

$$= \frac{9\pi\sqrt{65}}{16}\left\{\frac{144}{\sqrt{65}} + \frac{256}{65}\log\left(4+\frac{36}{\sqrt{65}}\right) + \frac{144}{\sqrt{65}} - \frac{256}{65}\log\left(-4+\frac{36}{\sqrt{65}}\right)\right\},$$

$$= 162\pi + \frac{144\pi}{\sqrt{65}}\log\frac{9+\sqrt{65}}{9-\sqrt{65}}.$$

Also solved by A. M. HARDING, NELLIE L. INGALS, HORACE OLSON, C. C. YEN, G. W. HARTWELL, H. C. FEEMSTER, J. A. ECKSON, GEORGE PAASWELL, O. S. ADAMS, and PAUL CAPRON.

MECHANICS.

321. Proposed by E. J. MOULTON, Northwestern University.

The attraction, A, in any direction, due to a homogeneous sphere, on a particle at the center of the sphere, using the Newtonian law, is obviously zero. Find the error in the following method of computing A. Take cylindrical coördinates with origin at the center of the sphere; let the Z-axis extend in the direction of the attraction to be computed, and let r, θ be the polar coördinates used. Let δ be the density and R the radius of the sphere, and k the constant of gravitation. Then

$$A = \int_{z=-R}^{z=R}\int_{r=0}^{r=\sqrt{R^2-z^2}}\int_{\theta=0}^{\theta=2\pi}\frac{k\delta r\,dz\,d\theta\,dr}{[r^2+z^2]^{\frac{3}{2}}} \tag{1}$$

$$= 2\pi k\delta\int_{z=-R}^{z=R}\left[\frac{-z}{(r^2+z^2)^{\frac{1}{2}}}\right]_{r=0}^{z=\sqrt{R^2-z^2}} dz \tag{2}$$

$$= 2\pi k\delta\int_{-R}^{R}\left[\frac{-z}{R}+1\right]dz \tag{3}$$

$$= 4\pi k\delta R. \tag{4}$$